Probability & Statistics

Chapter 4

Probability

Cambridge AS level

4.2 MUTUALLY EXCLUSIVE EVENTS AND THE ADDITION LAW

To find the probability that event A or event B occurs, we can simply add the probabilities of the two events together, but only if A *and* B are **mutually exclusive.**

Mutually exclusive events have **no common favourable outcomes**, which means that it is not possible for both events to occur, so $\mathbf{P(A \text{ and } B) = 0}$

Example 1:

When we roll an ordinary die, the events $even\ number = \{2; 4; 6\}$ and $factor\ of\ 5 = \{1; 5\}$ are mutually exclusive because they have no common outcomes.

We can that the intersection of these two sets is empty, Therefore

$P(even\ or\ factor\ of\ 5) = P(even) + P(factor\ of\ 5)$

4.2 MUTUALLY EXCLUSIVE EVENTS AND THE ADDITION LAW

Events are **not mutually exclusive** if they have **at least one common favourable outcome**, which means that it is possible for both events to occur, so $\boldsymbol{P(A\ and\ B) \neq 0}$

Example 2:

When we roll an ordinary die, the events $odd\ number = \{1;3;5\}$ and $factor\ of\ 5 = \{1;5\}$ are not mutually exclusive because they have common favourable outcomes.

We can that the intersection of these two sets is not empty, Therefore

$P(odd\ or\ factor\ of\ 5) \neq P(odd) + P(factor\ of\ 5)$

4.2 MUTUALLY EXCLUSIVE EVENTS AND THE ADDITION LAW

KEY POINT:

The addition law for mutually exclusive events is

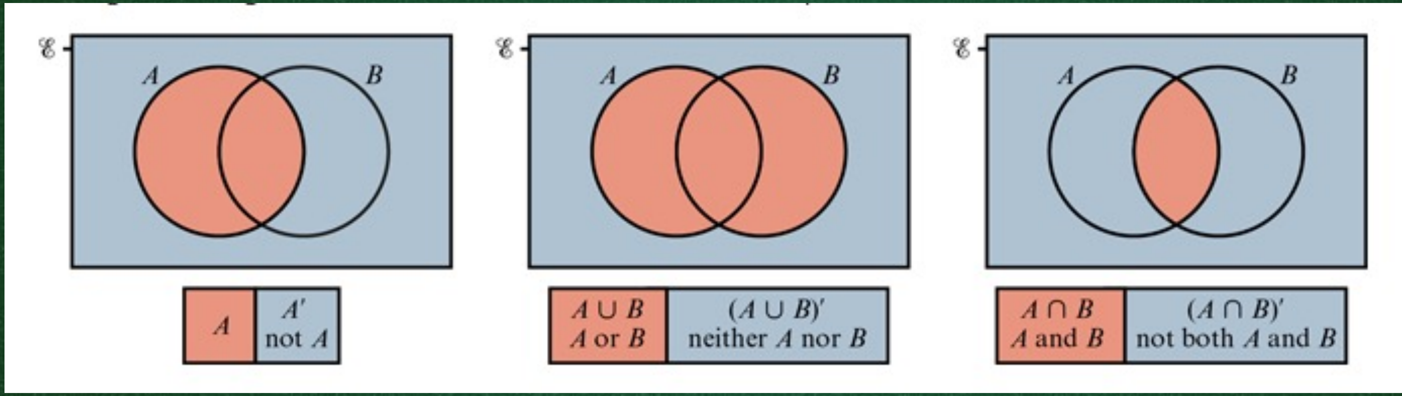
$$P(A \text{ or } B) = P(A) + P(B).$$

This can be extended for any number of mutually exclusive events:

$$P(A \text{ or } B \text{ or } C \text{ or } \ldots) = P(A) + P(B) + P(C) + \cdots$$

Venn diagrams

Venn diagrams are useful tools for solving problems in probability. We can use them to show favourable outcomes or the number of favourable outcomes or the probabilities of particular events.



Venn diagrams

TIP: The universal set ξ represents the complete set of outcomes and is called the *possibility space*.

TIP: A or B means event A occur or event B occurs or both occur.
$A \cup B$ means A or B and **$A \cap B$ means A and B**.

KEY POINT:
Using set notation, the addition law for mutually exclusive events is $\mathrm{P}(A \cup B) = P(A) + P(B)$.

A and B are mutually exclusive when $\mathrm{P}(A \cap B)=0$ that is, when $\mathrm{A} \cap B = \emptyset$ ($\emptyset$ −empty set)

Example 3: One digit is randomly selected from 1, 2, 3, 4, 5, 6, 7, 8 and 9. Three possible events are:

A: a multiple of 3 is selected

B: a factor of 8 is selected

C: a prime number is selected.

a. Show that the only fair of mutually events from *A*, *B* and *C* is *A* and *B*, and find $P(A \cup B)$.

b. Find

   a. $P(A \cup C)$

   b. $P(B \cup C)$.

$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{3, 6, 9\}\ P(A) = \frac{3}{9}$

$B = \{1, 2, 4, 8\}\ P(B) = \frac{4}{9}$

$C = \{2, 3, 5, 7\}\ P(C) = \frac{4}{9}$

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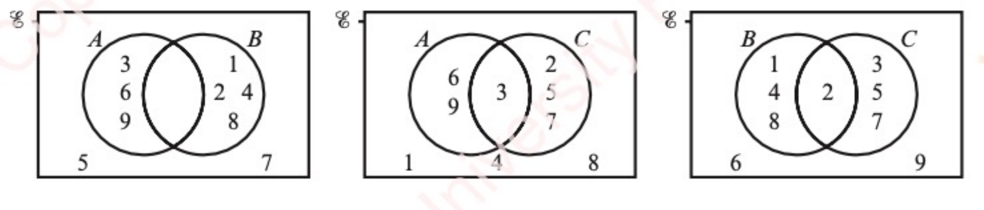

a. $A \cap B = \emptyset$, so *A* and *B* are mutually exclusive.

$A \cap C \neq \emptyset$, so *A* and *C* are not mutually exclusive.

$B \cap C \neq \emptyset$, so *B* and *C* are not mutually exclusive.

$$P(A \cup B) = P(A \text{ or } B) = \frac{3}{9} + \frac{4}{9} = \frac{7}{9}$$

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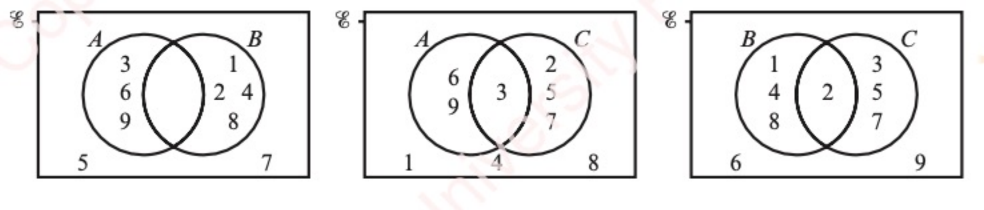
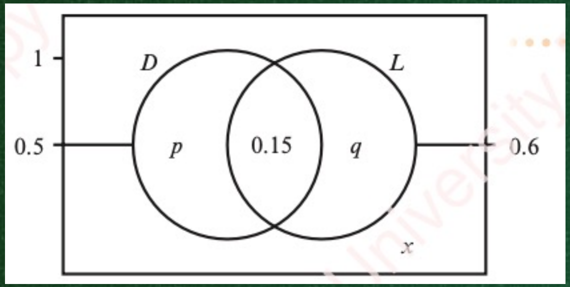


$$P(A \cup C) = P(A) + P(C) - P(A \cap C) = \frac{3}{9} + \frac{4}{9} - \frac{1}{9} = \frac{6}{9}$$

$$P(B \cup C) = P(B) + P(C) - P(B \cap C) = \frac{4}{9} + \frac{4}{9} - \frac{1}{9} = \frac{7}{9}$$

Example 4: In a survey 50% of the participants own a desktop (D), 60% own a laptop (L) and 15% own both. What percentage of the participants owns neither a desktop nor a laptop?



$p = 0.5 - 0.15 = 0.35$

$q = 0.6 - 0.15 = 0.45$

$x = 1 - (0.35 + 0.15 + 0.45) = 0.05$

5% of the participants own neither a desktop nor a laptop.

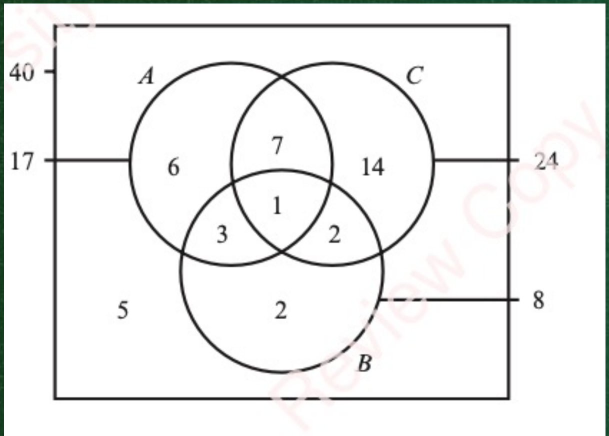
$P(D \cup L) = P(D) + P(L) - P(D \cap L) = 0.5 + 0.6 - 0.15 = 0.95$

$1 - P(D \cup L) = 1 - 0.95 = 0.05$

Example 5: Forty children were each asked which fruits they like from apples (A), bananas (B) and cherries (C).

The following Venn diagram shows the number of children that like each type of fruit.

Find the probability that a randomly selected child likes apples or bananas.



$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{17}{40} + \frac{8}{40} - \frac{4}{40} = \frac{21}{40}$$

4.2 MUTUALLY EXCLUSIVE EVENTS AND THE ADDITION LAW

KEY POINT: For any two events, A and B, P(A or B) $=$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

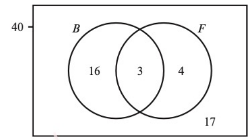
Homework Page 98 – Exercise 4B

EXERCISE 4B

1 Find the probability that the number rolled with an ordinary fair die is:

a a prime number or a 4 b a square number or a multiple of 3 c more than 3 or a factor of 8.

2 A group of 40 students took a test in Economics. The following Venn diagram shows that 19 boys (B) took the test and that seven students failed the test (F).



a Describe the 21 students who are members of the set B'.

b Find the probability that a randomly selected student is a boy or someone who failed the test.

3 The following table gives information about all the animals on a farm.

	Male	Female
Goats	5	25
Sheep	3	22

a Find the probability that a randomly selected animal is:

i male or a goat ii a sheep or female.

b Find a different way of describing each of the two types of animal in part **a**.

4 Two ordinary fair dice are rolled and three events are:

X: the sum of the two numbers rolled is 6.

Y: the difference between the two numbers rolled is zero.

Z: both of the numbers rolled are even.

a List the outcomes that are favourable to:

i X and Y ii X and Z iii Y and Z.

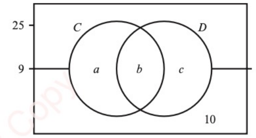
b What do your answers to part **a** tell you about the events X, Y and Z?

5 The letters A, B, B, B, C, D, D and E are written onto eight cards and placed in a bag. Find the probability that the letter on a randomly selected card is:

a a vowel or in the word DOMAIN

b a consonant or in the word DOUBLE.

6 In a group of 25 boys, nine are members of the chess club (C), eight are members of the debating club (D) and 10 are members of neither of these clubs. This information is shown in the Venn diagram.



a Find the values of a, b and c.

b Find the probability that a randomly selected boy is:

i a member of the chess club or the debating club

ii a member of exactly one of these clubs.

7 Forty girls were asked to name the capital of Cuba and of Hungary; 19 knew the capital of Cuba, 20 knew the capital of Hungary and seven knew both.

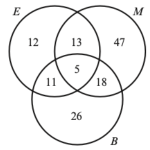
a Draw a Venn diagram showing the number of girls who knew each of these capitals.

b Find the probability that a randomly selected girl knew:

i the capital of Cuba but not of Hungary ii just one of these capitals.

8 In a survey on pet ownership, 36% of the participants own a cat, 20% own a hamster but not a cat, and 8% own a hamster and a cat. What percentage of the participants owns neither a hamster nor a cat?

9 A garage repaired 132 vehicles last month. The number of vehicles that required electrical (E), mechanical (M) and bodywork (B) repairs are given in the diagram opposite.



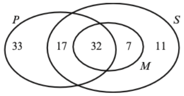
Find the probability that a randomly selected vehicle required:

a mechanical or bodywork repairs

b no bodywork repairs

c exactly two types of repair.

10 The 100 students at a technical college must study at least one subject from Pure Mathematics (P), Statistics (S) and Mechanics (M). The numbers studying these subjects are given in the diagram opposite.



a Who does the number 17 in the diagram refer to?

b Find the probability that a randomly selected student studies:

i Pure Mathematics or Mechanics

ii exactly two of these subjects.

c List the three subjects in ascending order of popularity.

11 Events X and Y are such that $P(X) = 0.5$, $P(Y) = 0.6$ and $P(X \cap Y) = 0.2$.

a State, giving a reason, whether events X and Y are mutually exclusive.

b Using a Venn diagram, or otherwise, find $P(X \cup Y)$.

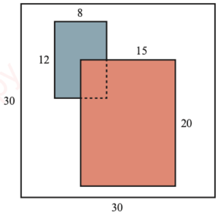
c Find the probability that X or Y, but not both, occurs.

12 A, B and C are events where $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.3$, $P(A \cap B) = 0.12$, $P(A \cap C) = 0$ and $P(B \cap C) = 0.1$.

a State which pair of events from A, B and C is mutually exclusive.

b Using a Venn diagram, or otherwise, find $P[(A \cup B \cup C)']$, which is the probability that neither A nor B nor C occurs.

13 The diagram opposite shows a 30 cm square board with two rectangular cards attached. The 15 cm by 20 cm card covers one-quarter of the 8 cm by 12 cm card.



A dart is randomly thrown at the board, so that it sticks within its perimeter. Use areas to calculate the probability that the dart pierces:

a both cards

b at least one of the cards

c exactly one of the cards.

Homework

Page 98 – Exercise 4B

14 Given that $P(A) = 0.4$, $P(B) = 0.7$ and that $P(A \cup B) = 0.8$, find:

a $P(A \cup B')$ **b** $P(A' \cap B)$

15 Each of 27 tourists was asked which of the countries Angola (A), Burundi (B) and Cameroon (C) they had visited. Of the group, 15 had visited Angola; 8 had visited Burundi; 12 had visited Cameroon; 2 had visited all three countries; and 21 had visited only one. Of those who had visited Angola, 4 had visited only one other country. Of those who had not visited Angola, 5 had visited Burundi only. All of the tourists had visited at least one of these countries.

a Draw a fully labelled Venn diagram to illustrate this information.

b Find the number of tourists in set B' and describe them.

c Describe the tourists in set $(A \cup B) \cap C'$ and state how many there are.

d Find the probability that a randomly selected tourist from this group had visited at least two of these three countries.