# PRACTICAL MAXIMUM AND MINIMUM PROBLEMS

As level

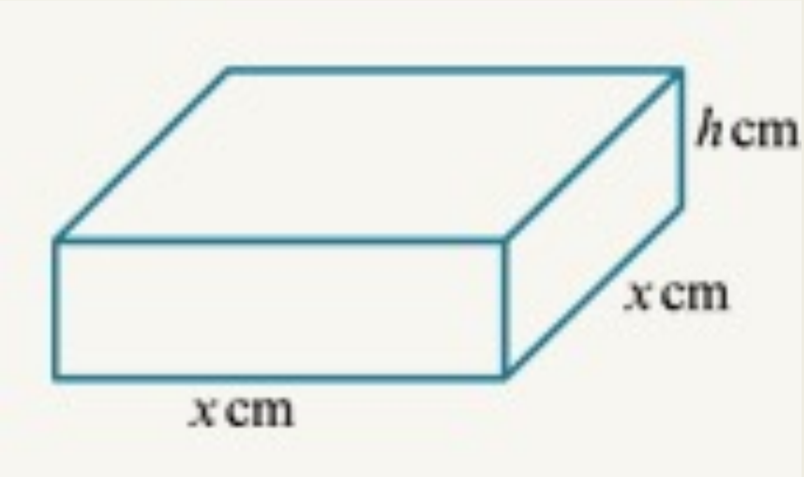
Example 1:

The surface area of the solid cuboid is $100cm^2$ and the volume is $V\ cm^3$.

a. Express $h$ in terms of $x$.

b. Show that $V = 25x - \frac{1}{2}x^3$.

c. Given that $x$ can vary, find the stationary value of $V$ and determine whether this stationary value is a maximum or a minimum.




Surface area=$2xh + 2xh + 2x^2$

Surface area=$4xh + 2x^2$

$$4xh + 2x^2 = 100cm^2$$

$$4xh = 100 - 2x^2$$

$$h = \frac{100}{4x} - \frac{2x^2}{4x}$$

$$\boldsymbol{h = \frac{25}{x} - \frac{x}{2}}$$

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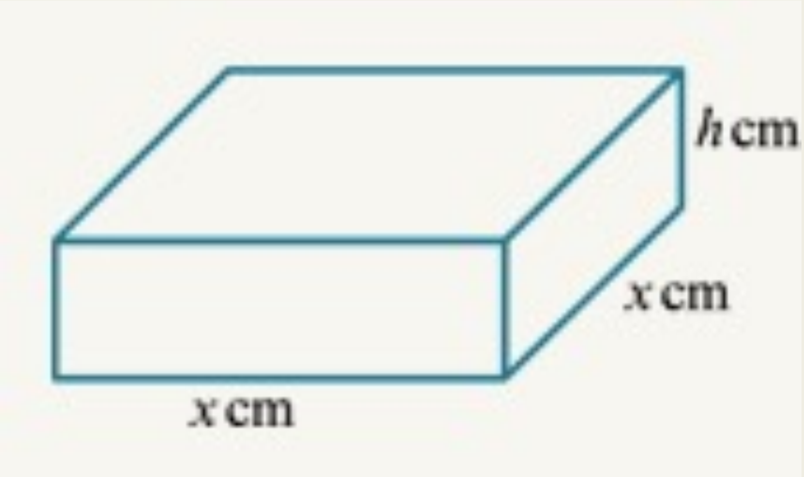
Volume=$x^2h$

$$\boldsymbol{h = \frac{25}{x} - \frac{x}{2}}$$

Volume=$x^2\left(\frac{25}{x} - \frac{x}{2}\right)$

**Volume=$25x - \frac{x^3}{2}$**



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c. Given that $x$ can vary, find the stationary value of $V$ and determine whether this stationary value is a maximum or a minimum.

**Volume=$25x-\frac{x^3}{2}$**

$$\frac{dv}{dx}=25-\frac{3}{2}x^2$$

$$25-\frac{3}{2}x^2=0$$

$$x^2=\frac{50}{3}$$

$$x=\pm\sqrt{\frac{50}{3}}=\pm\frac{5\sqrt{2}}{\sqrt{3}}=\pm\frac{5\sqrt{6}}{3}$$

$$x=\frac{5\sqrt{6}}{3}$$

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$$Volume=25\times\frac{5\sqrt{6}}{3}-\frac{1}{2}\times\left(\frac{5\sqrt{6}}{3}\right)^3$$

$$\frac{d^2v}{dx^2}=-3x$$

$$x=\frac{5\sqrt{6}}{3}$$

$$\frac{d^2v}{dx^2}=-3\times\frac{5\sqrt{6}}{3}=-5\sqrt{6}<0$$

Max

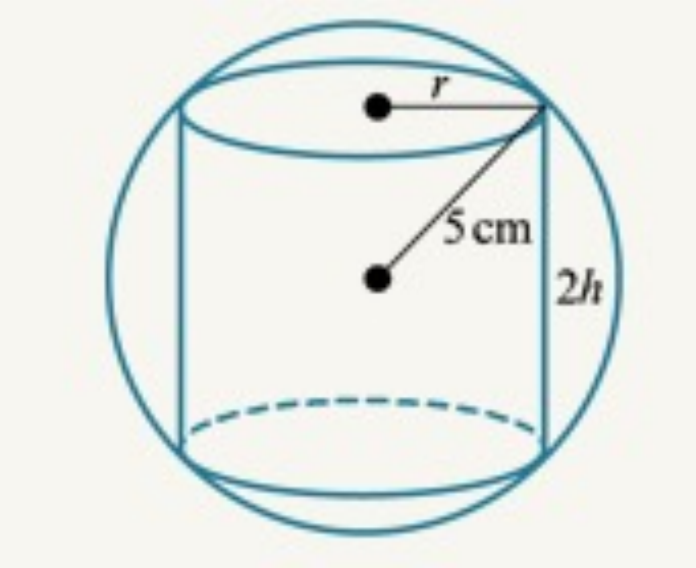
**Example 2:** The diagram shows a solid cylinder of radius $r$ cm and height $2h\ cm$ cut from a solid sphere of radius $5\ cm$. The volume of the cylinder is $C\ cm^3$.

a. Express $r$ in terms of $h$.

b. Show that $V = 50\pi h - 2\pi h^3$

c. Find the value for $h$ for which there is stationary value of $V$.

d. Determine the nature of this stationary value.




$$r^2 + h^2 = 25$$

$$r^2 = 25 - h^2$$

$$\boldsymbol{r = \sqrt{25 - h^2}}$$

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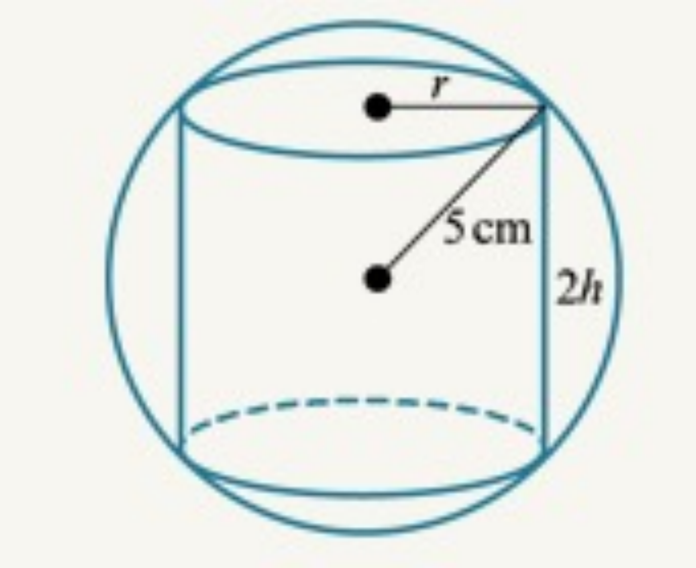
$V = \pi r^2 h$

$C = \pi r^2 \times 2h$

$C = \pi(\sqrt{25 - h^2})^2 \times 2h$

$C = \pi(25 - h^2) \times 2h$

$C = 50h\pi - 2\pi h^3$



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$$\boldsymbol{V = 50h\pi - 2\pi h^3}$$

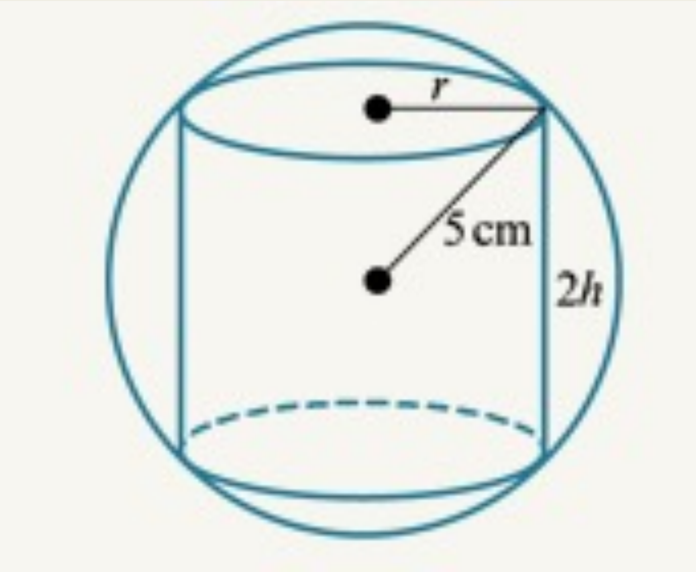
$$\frac{dV}{dh} = 50\pi - 6\pi h^2$$

$$50\pi - 6\pi h^2 = 0$$

$$50\pi = 6\pi h^2$$

$$h^2 = \frac{50}{6} = \frac{25}{3}$$

$$h = \sqrt{\frac{25}{3}} = \frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$$



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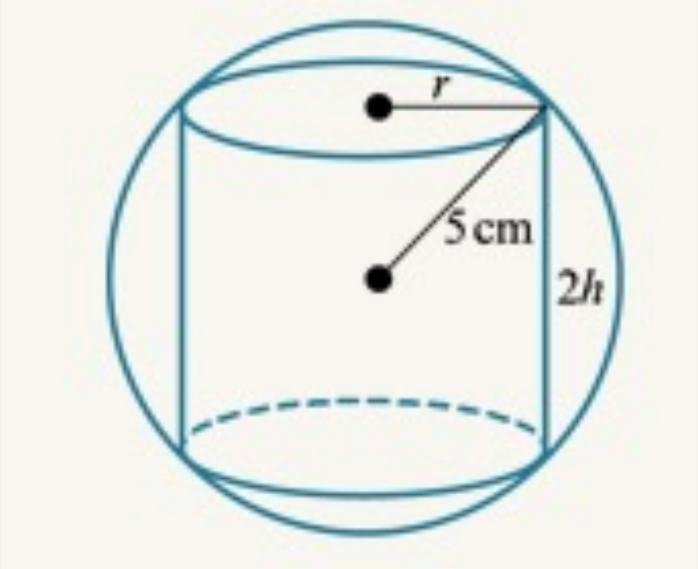



$$\frac{dV}{dh} = 50\pi - 6\pi h^2$$

$$\frac{d^2V}{dh^2} = -12\pi h$$

$$h = \frac{5\sqrt{3}}{3} \qquad \frac{d^2V}{dh^2} = -12\pi \times \frac{5\sqrt{3}}{3} = -20\sqrt{3}\pi < 0$$

Max

Example 3: The diagram shows a hollow cone with base radius $12\ cm$ and height $24\ cm$.

A solid cylinder stands on the base of the cone and upper edge touches the inside of the cone. The cylinder has base radius $r\ cm$, height $h\ cm$ and volume $V\ \text{cm}^3$.

a. Express $h$ in terms of $r$.

b. Show that $V = 24\pi r^2 - 2\pi r^3$.

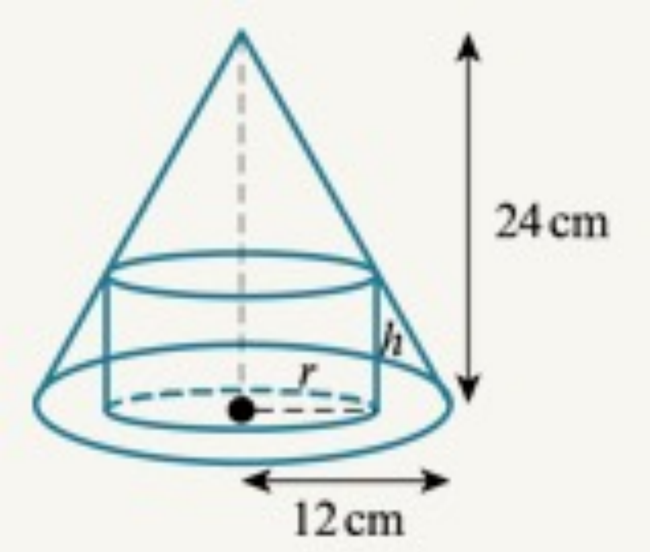
c. Find the volume of the largest cylinder that can stand inside the cone.

$$\frac{12}{r} = \frac{24}{24 - h}$$

$$\boldsymbol{h = 24 - 2r}$$

$$12(24 - h) = 24r$$

$$24 - h = 2r$$



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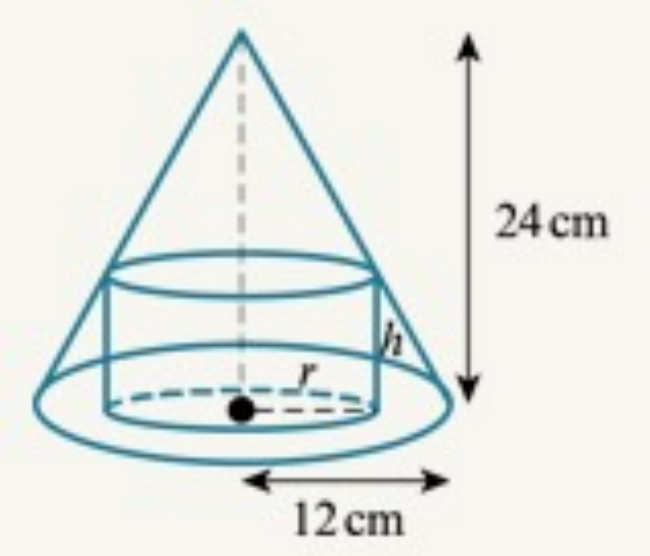
$V = \pi r^2 h$

$\boldsymbol{h = 24 - 2r}$

$V = \pi r^2 (24 - 2r)$

$\boldsymbol{V = 24\pi r^2 - 2\pi r^3}$



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$$\boldsymbol{V = 24\pi r^2 - 2\pi r^3}$$

$$\frac{dV}{dr} = 48r\pi - 6r^2\pi$$

$$48r\pi - 6r^2\pi = 0$$

$$6r\pi(8 - r) = 0$$

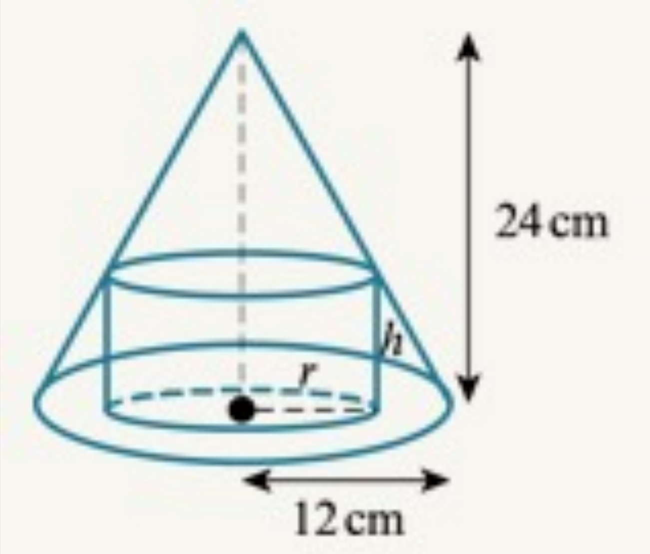
$$r_1 = 0 \qquad r_2 = 8$$

$$\frac{d^2V}{dr^2} = 48\pi - 12r\pi$$

$$r = 8$$

$$\frac{d^2V}{dr^2} = 48\pi - 12\times 8\pi$$

$$\frac{d^2V}{dr^2} = -48\pi < 0 \qquad \text{Max}$$



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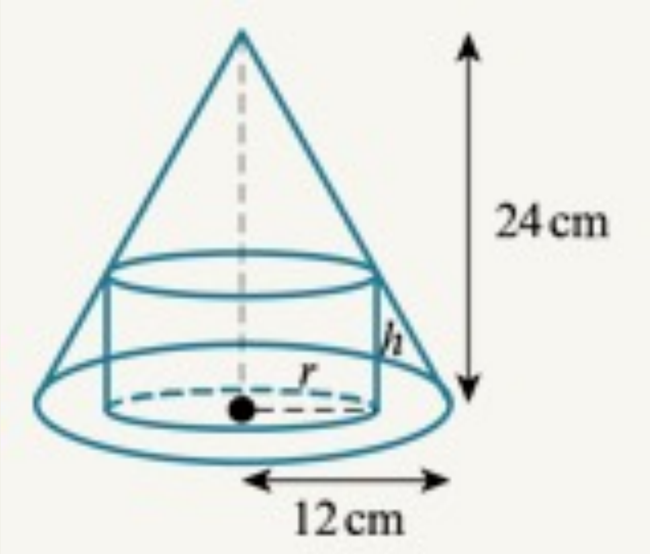


$$\boldsymbol{V = 24\pi r^2 - 2\pi r^3}$$

$$r = 8$$

$$V = 24\pi \times 8^2 - 2\pi \times 8^3$$

$$V = 512\pi$$

Max