

Completing the square

□ Example 1:

Express $x^2 + 10x + 32$ in completed square form

$$\begin{aligned}x^2 + 10x + 32 &= (x^2 + 10x) + 32 = \\ &= (x^2 + 10x + 25) - 25 + 32 = \mathbf{(x + 5)^2 + 7};\end{aligned}$$

$$(x + 5)(x + 5) = x^2 + 10x + 25$$

□ Example 2:

Express $2x^2 + 10x + 7$ in completed square form

$$2x^2 + 10x + 7 = 2(x^2 + 5x) + 7 = 2\left(x^2 + 5x + \frac{25}{4} - \frac{25}{4}\right) + 7 =$$

$$2\left(x^2 + 5x + \frac{25}{4}\right) - \frac{25}{2} + 7 = \mathbf{2\left(x + \frac{5}{2}\right)^2 - \frac{11}{2}};$$

$$\left(x + \frac{5}{2}\right)\left(x + \frac{5}{2}\right) = x^2 + 5x + \frac{25}{4}$$

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□ Example 3:

Express $3 - 4x - 2x^2$ in completed square form

$$\begin{aligned} 3 - 4x - 2x^2 &= -2x^2 - 4x + 3 = -2(x^2 + 2x) + 3 = \\ &= -2(x^2 + 2x + 1 - 1) + 3 = -2(x^2 + 2x + 1) + 2 + 3 = \\ &= \mathbf{-2(x + 1)^2 + 5}; \end{aligned}$$

$$(x + 1)(x + 1) = x^2 + 2x + 1$$

□ Example 4:

Express $12x^2 - 7x - 12$ in completed square form

$$\begin{aligned} 12x^2 - 7x - 12 &= (12x^2 - 7x) - 12 = 12\left(x^2 - \frac{7}{12}x\right) - 12 = \\ &= 12\left(x^2 - \frac{7}{12}x + \frac{49}{576} - \frac{49}{576}\right) - 12 = 12\left(x - \frac{7}{24}\right)^2 - \frac{49}{48} - 12 = \\ &= 12\left(x - \frac{7}{24}\right)^2 - \frac{625}{48} = \mathbf{12\left(x - \frac{7}{24}\right)^2 - 13\frac{1}{48}}; \end{aligned}$$

$$\begin{aligned} &\left(x - \frac{7}{24}\right)\left(x - \frac{7}{24}\right) \\ &= x^2 - \frac{7}{12}x + \frac{49}{576} \end{aligned}$$

□ Exercise 1

Express $x^2 + 8x - 1$ in completed square form

$$\begin{aligned} x^2 + 8x - 1 &= (x^2 + 8x) - 1 = \\ &= (x^2 + 8x + 16) - 16 - 1 = \mathbf{(x + 4)^2 - 17}; \end{aligned}$$

$$(x + 4)(x + 4) = x^2 + 8x + 16$$

□ Exercise 2

Express $x^2 - 10x + 3$ in completed square form

$$\begin{aligned} x^2 - 10x + 3 &= (x^2 - 10x) + 3 = \\ (x^2 - 10x + 25) - 25 + 3 &= \mathbf{(x - 5)^2 - 22} \end{aligned}$$

$$(x - 5)(x - 5) = x^2 - 10x + 25$$

□ Exercise 3

Express $x^2 + 3x - 2$ in completed square form

$$\begin{aligned} x^2 + 3x - 2 &= (x^2 + 3x) - 2 = \left(x^2 + 3x + \frac{9}{4}\right) - \frac{9}{4} - 2 = \\ &= \mathbf{\left(x + \frac{3}{2}\right)^2 - \frac{17}{4}}; \end{aligned}$$

$$\left(x + \frac{3}{2}\right)\left(x + \frac{3}{2}\right) = x^2 + 3x + \frac{9}{4}$$

• Exercise4

Express $x^2 - 5x + 1$ in completed square form

$$x^2 - 5x + 1 = (x^2 - 5x) + 1 = \left(x^2 - 5x + \frac{25}{4}\right) - \frac{25}{4} + 1 =$$

$$= \left(x - \frac{5}{2}\right)^2 - \frac{21}{4};$$

$$\left(x - \frac{5}{2}\right)\left(x - \frac{5}{2}\right) = x^2 - 5x + \frac{25}{4}$$

Exercise5

Express $3x^2 - 12x + 5$ in completed square form

$$2x^2 + 10x + 7 = 2(x^2 + 5x) + 7 = 2\left(x^2 + 5x + \frac{25}{4} - \frac{25}{4}\right) + 7 =$$

$$2\left(x^2 + 5x + \frac{25}{4}\right) - \frac{25}{2} + 7 = 2\left(x + \frac{5}{2}\right)^2 - \frac{11}{2};$$

$$\left(x - \frac{7}{24}\right)\left(x - \frac{7}{24}\right) = x^2 - \frac{7}{12}x + \frac{49}{576}$$

